for a better future

KEP

Gear Ⅰ - Spur Gears

R&D Center

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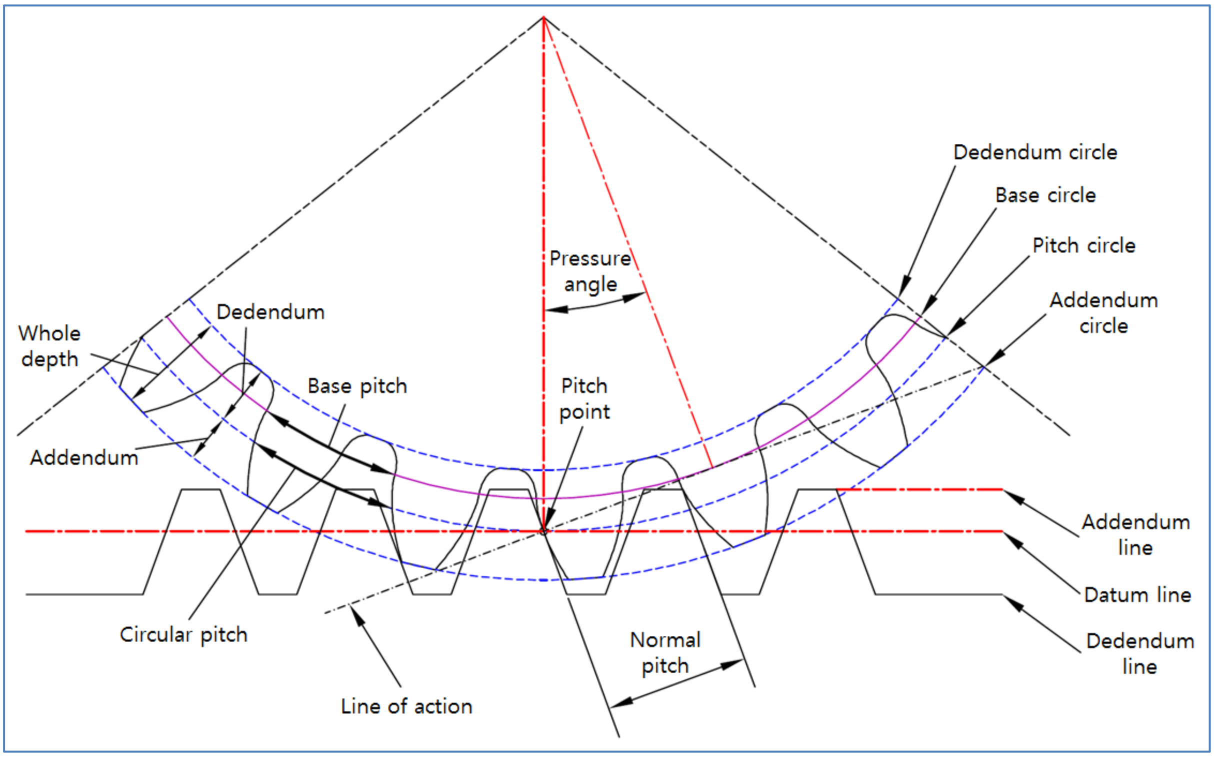
KOREA ENGINEERING PLASTICS CO.,LTD.

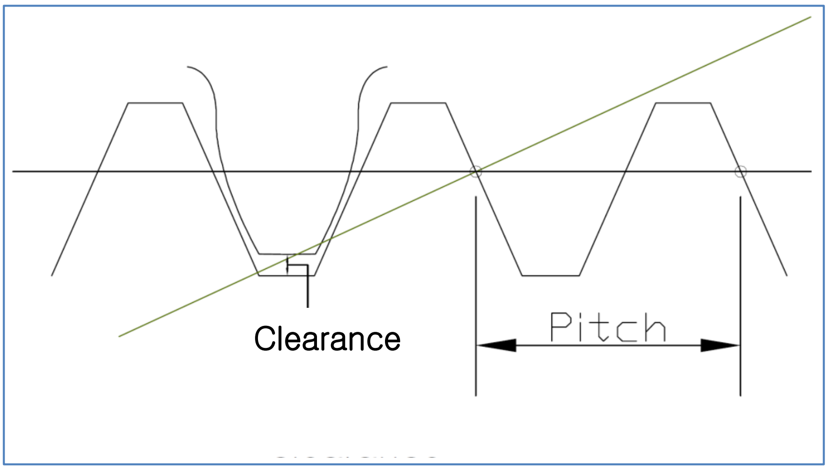
1. Standard of gear (KS)

(1) Terms of gear KS B 0102
(2) Standard of expression and geometrical data of gear KS B 0053
(3) Technical drawings - Representations of gears KS B 0002

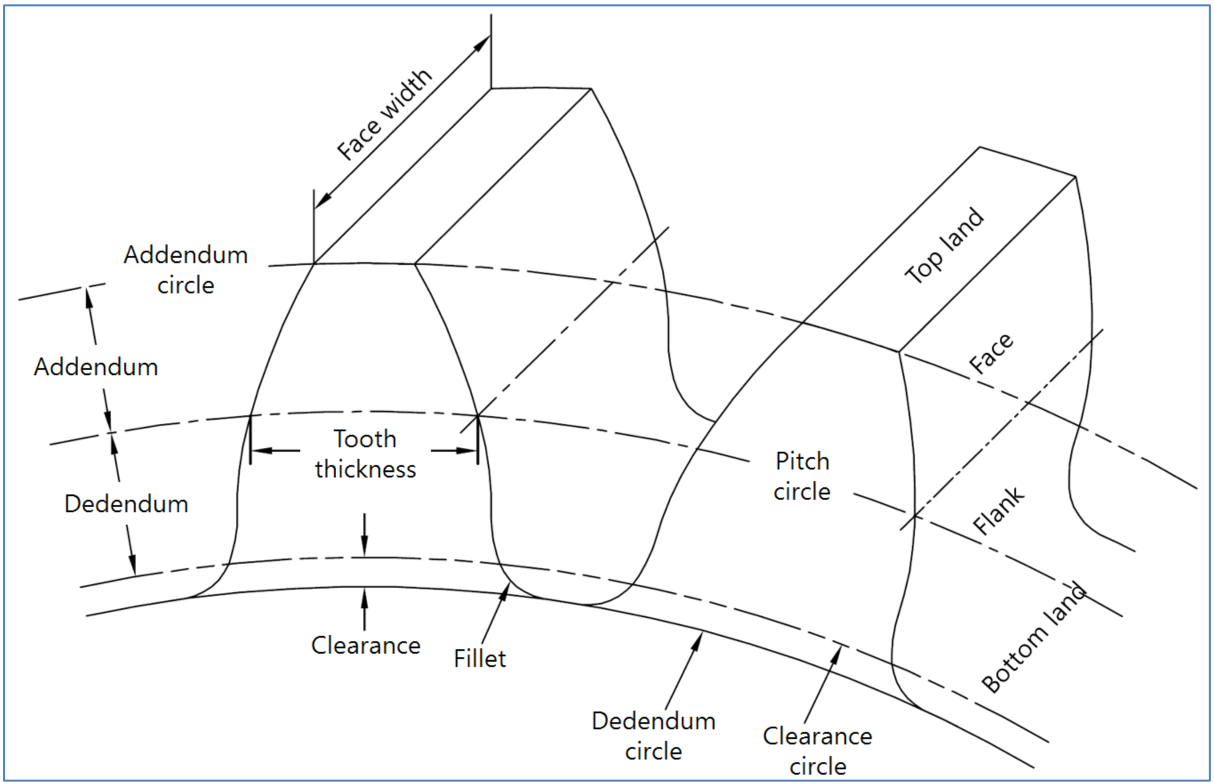
2. Terms of gear



[Figure 1] Terms of gear



[Figure 2] Clearance



[Figure 3] Terms of gear teeth

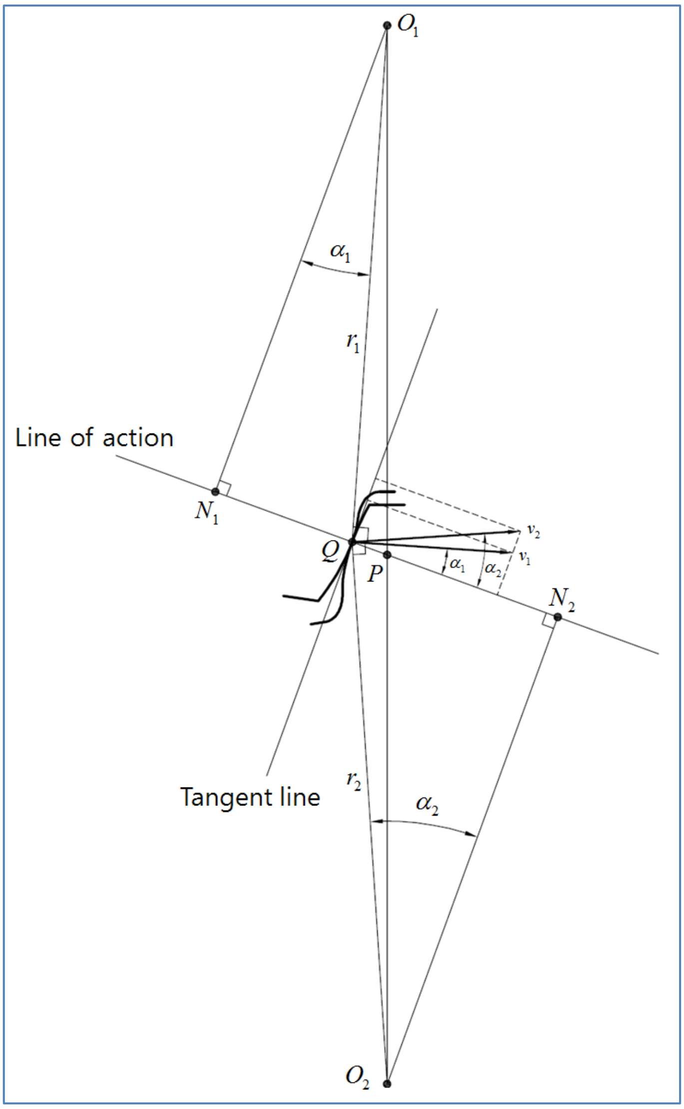
(1) Pitch point : a point where the line of action meets the line drawn between center of two gears, the point where pitch circles of two gears meet
(2) Pitch circle : a circle drawn through the pitch point from each gear center
(3) Base circle : a base circle where an involute curve is drawn
(4) Line of action : a common tangent line drawn at the plane of contact of the tooth plane when intermeshing gears rotate; it represents the direction of action of the forces.
(5) Pressure angle : an angle between the line of action and the common tangent line of pitch circle of engaging two gears
(6) Addendum circle : a circle drawn by connecting the top of gear teeth
(7) Dedendum circle : a circle drawn by connecting the root of gear teeth
(8) Addendum : distance between pitch circle and addendum circle
(9) Dedendum : distance between pitch circle and dedendum circle
(10) Circular pitch : an arc of a circle of a tooth at circumference of pitch circle, a circumference of pitch circle divided by tooth number, sum of an arc of a circle of pitch circle of tooth and tooth groove.
(11) Normal pitch : same value with base circle pitch, the distance between a tooth and an adjacent tooth on common tangent line
(12) Clearance : distance from a center of one addendum circle of a gear to a center of a dedendum circle of an opposing gear
(13) Whole depth : total height of tooth = addendum + dedendum
(14) Working depth : sum of addendum of a pair of gears

(14) Backlash : a gap between tooth surface
(Backlash on the circular pitch, backlash direction to the tangent line, backlash direction to the radius)

3. Basic theory of gear

3-1. Tooth profile of gear

(1) Condition of engagement of gears : When two gears engage, they should be in contact at a point on the tooth profile curve. To prevent separation or digging of tooth profiles, two gears should have the same velocity of a tangent line direction at contact Q.



[Figure 4] Tooth profile curve

(2) Theory of mechanics condition of tooth profile curve

* O_1, O_2 $\rightarrow$ Rotational center, fixed point

* The rate of gear velocity : Constant

- Velocity at the Q point of gear 1 $v_1 = r_1\omega_1$

- Velocity at the Q point of gear 2 $v_2 = r_2\omega_2$

- Velocity of a tangent line direction = $v_1 \cos\alpha_1 = v_2 \cos\alpha_2$
- $r_1\omega_1 \cos\alpha_1 = r_2\omega_2 \cos_2$
- $\frac{\omega_1}{\omega_2} = \frac{r_2 \cos\alpha_2}{r_1 \cos\alpha_1} = \frac{O_2N_2}{O_1N_1} = \frac{O_2P}{O_1P}$
- $\Delta O_1N_1P \propto \Delta O_2N_2P$ $\rightarrow$ Similar figures

* P point : The point which divides center distance $\overline{O_1O_2}$ internally ($\frac{\omega_1}{\omega_2}$) → fixed point → pitch point

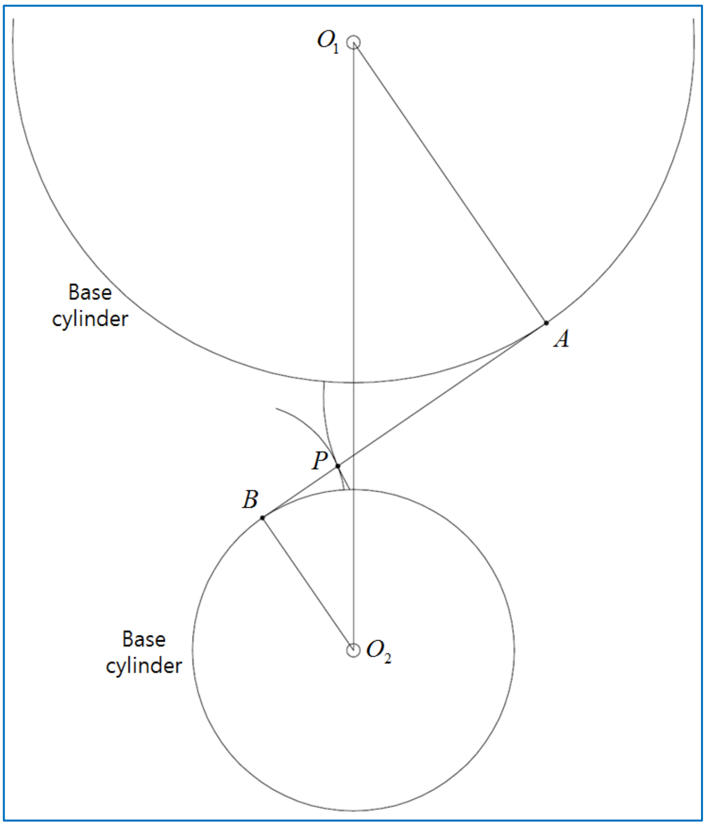
3-2. Pitch point

(1) Camus theorem : In a contact point, a common tangent line built at the tooth profile passes the pitch point.

① There are a lot of curves which satisfy the condition.

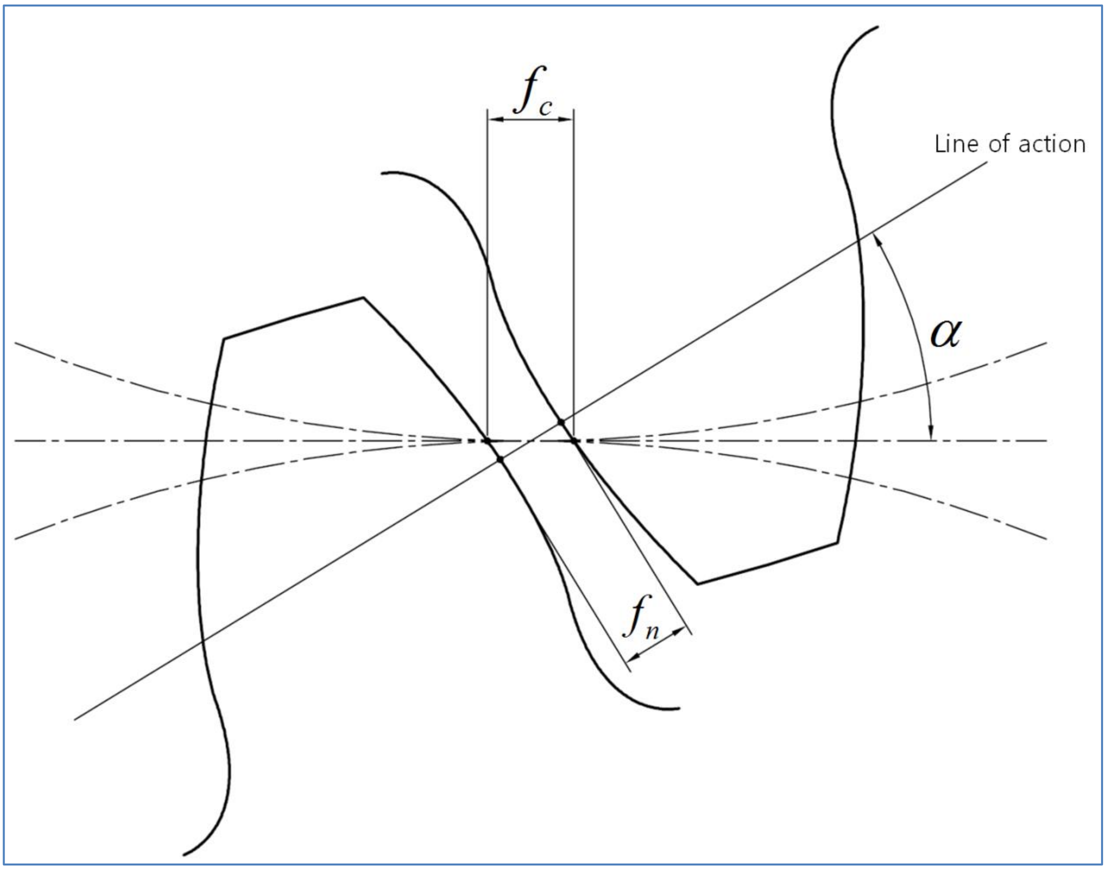
② We limit the number to 2 or 3 curves in the practical view.

③ Involute curve and Cycloid



[Figure 5] Principle of the Involute tooth profile

3-3. Backlash



[Figure 6] Backlash

(1) Two methods to make a backlash

① A method in which total tooth thickness at the circular pitch is as short as a backlash of circumference of circular pitch : We can divide the backlash length of gear engagement in half or we can determine the tooth thickness of the gear(drive shaft) p/2 and the tooth thickness of the gear(engaging) p/2-f.

② A method in which center distance is as long as f_r

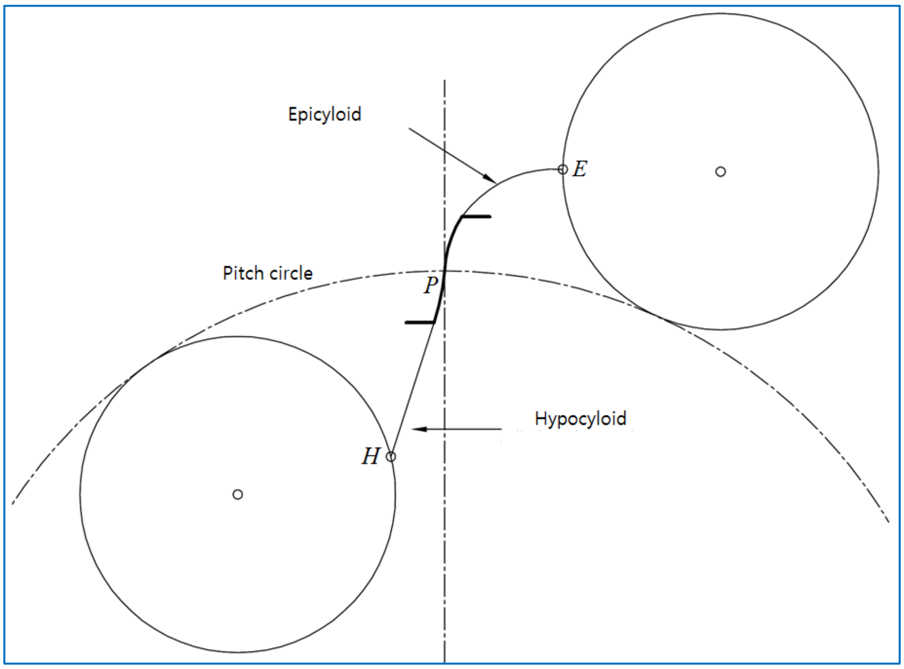
3-4. Cycloid tooth profile

(1) A trajectory of a point on the rolling circle when a small rolling circle rolls on the base circle without sliding.

(2) Cycloid curve

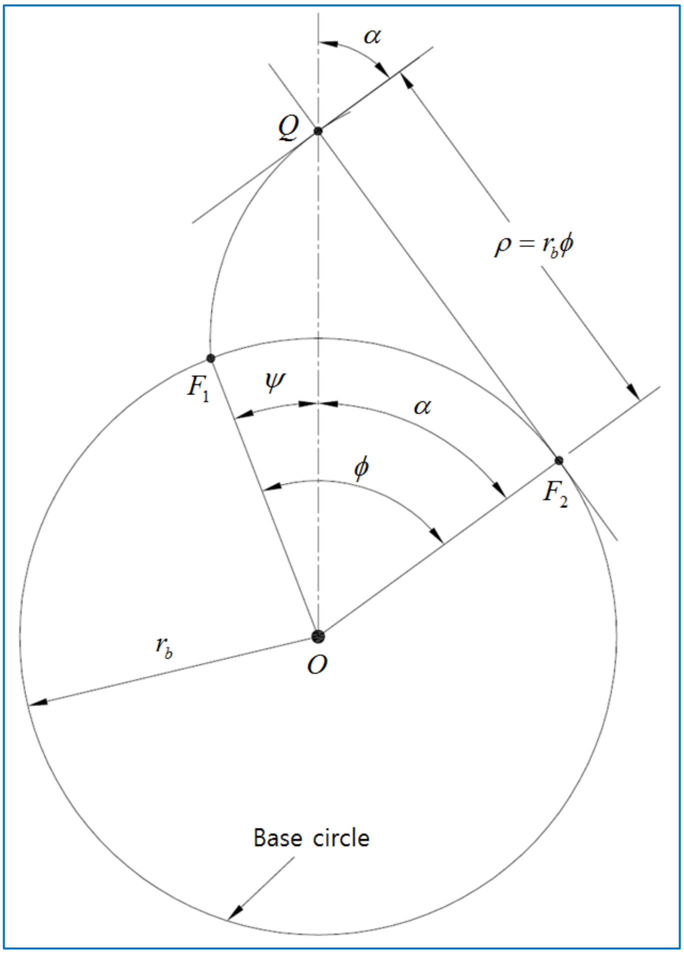
① Epicycloids curve : A curve drawn outside of the pitch circle

② Hypocycloid curve : A curve drawn inside of the pitch circle



[Figure 7] Cycloid tooth profile curve

3-5. Involute tooth profile



[Figure 8] Involute tooth profile curve

(1) $\overline{F_1F}$ is the radius of curvature ρ of involute curve.

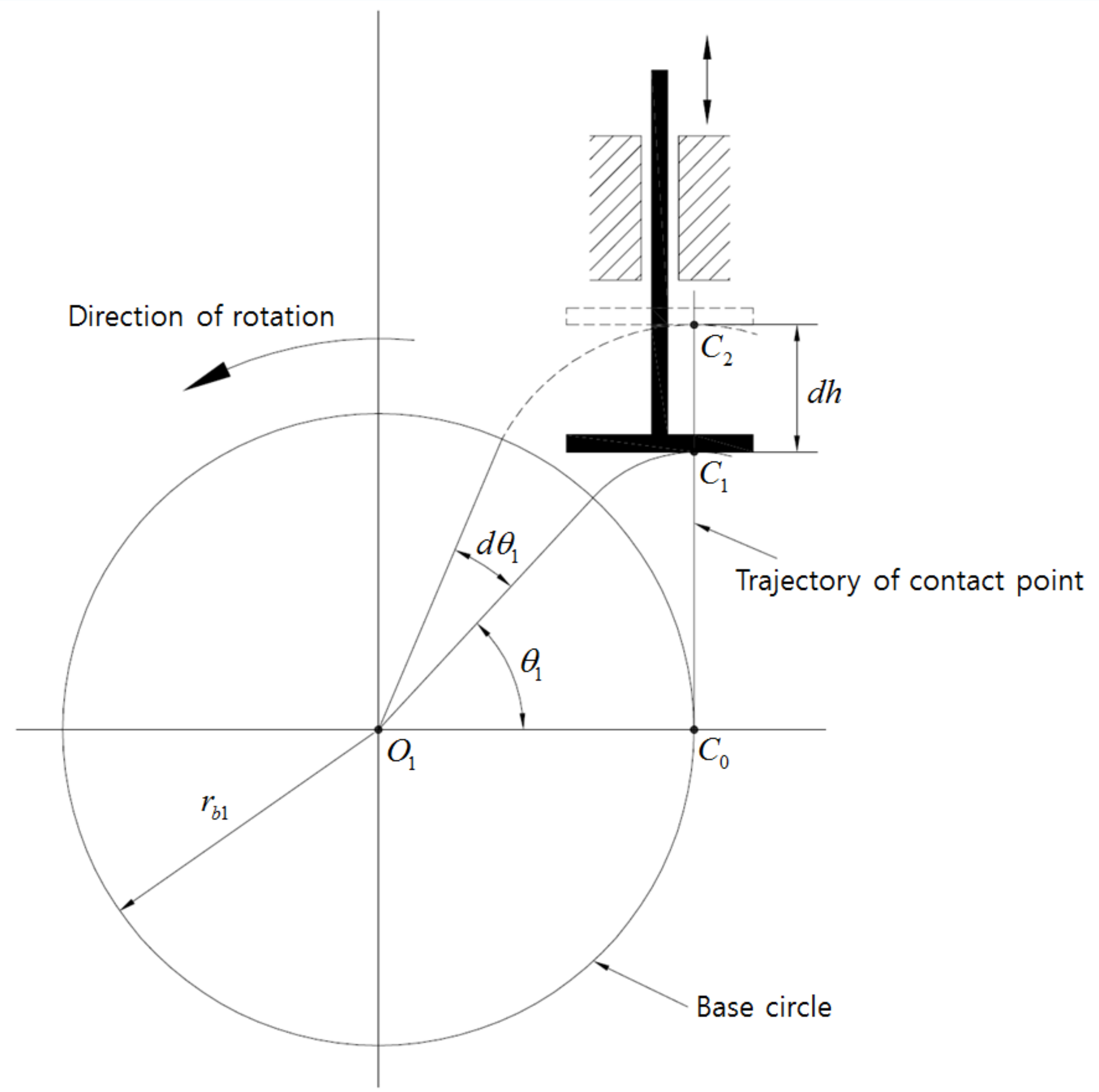
(2) Base circle → Involute tooth profile curve exists only outside of the base circle

3-6. Involute tooth profile and Cycloid tooth profile

[Table 1] Involute tooth profile and Cycloid tooth profile

	Involute tooth profile	Cycloid tooth profile
Pressure angle	Constant	Changing
Specific sliding/Abrasion	Specific sliding changes at the tooth surface and the tooth profile is collapses easily at the tooth top and the pitch circle. Specific sliding is 0 at the pitch point. / Unbalanced abrasion, change of the tooth profile	Specific sliding is same at all tooth surfaces and the tooth profile wears down uniformly. Uniform specific sliding is the best thing about cycloid gears.
Cutting tool	A straight line(trapezium). It is easy to design and the price is reasonable.	A cycloid curve. Cutter varies with rolling circle.
Design method	A few errors are acceptable due to space. A possible profile-shifted cutting.	An accurate size is required. An impossible profile-shifted cutting.
Center distance/ Assemble	The gears engage in theory of mechanism in spite of a few errors of center distance of the gear box / Easy	An accurate size is required or the gears won't engage in mechanism theory. There is possibility of tooth profile damage. / Difficult.
Undercut	O	X
Compatibility	A pressure angle and module should be the same.	A circular pitch and rolling circle should be the same.
Application	Motors, general applications	Precision instruments(watches, instruments)
Intermeshing	Applied pressure is high because of intermeshing between relief planes.	Applied stress is lower than an involute tooth profile because of intermeshing between the relief plane and intaglio plane.

3-7. Engagement of Involute tooth profile



[Figure 9] Properties of an Involute tooth profile

(1) Properties of Involute tooth profile curve

- r_{b1} : Radius of a base circle
- Search for a plate that contacts with an Involute tooth profile (plate goes up and down)

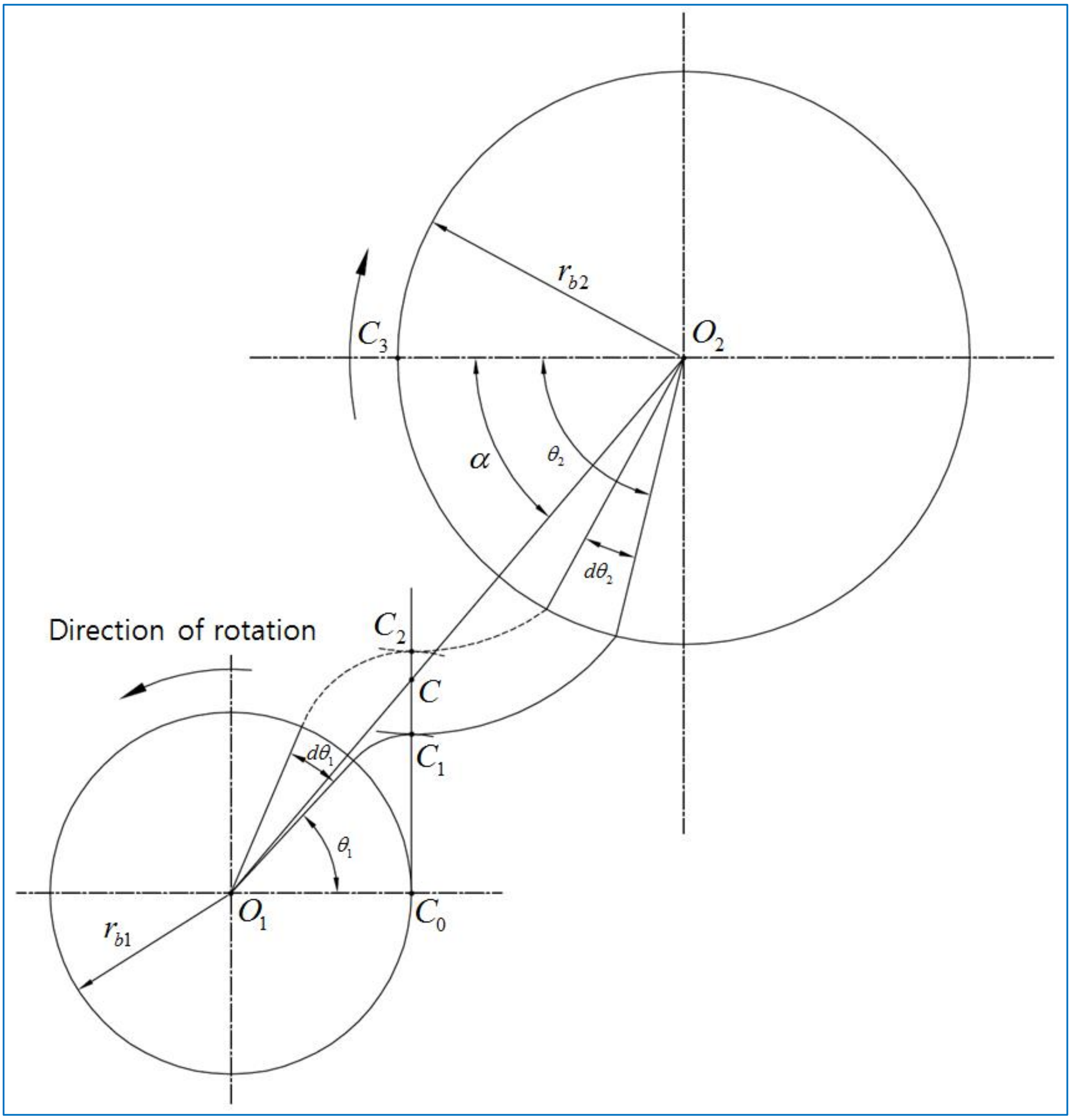
$$dh = \overline{C_1C_2} = r_{b1} \times d\theta$$

- Velocity v

Differentiate by time

$$v = \frac{dh}{dt} = r_{b1}\frac{d\theta}{dt} = r_{b1}\omega$$

* Gear rotates at a constant velocity → A slab is lifted at a constant velocity → Uniform motion

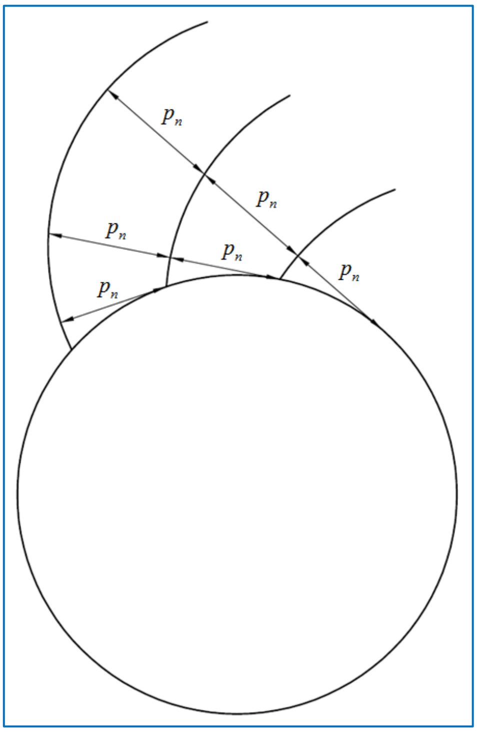


[Figure 10] Properties of an Involute tooth profile

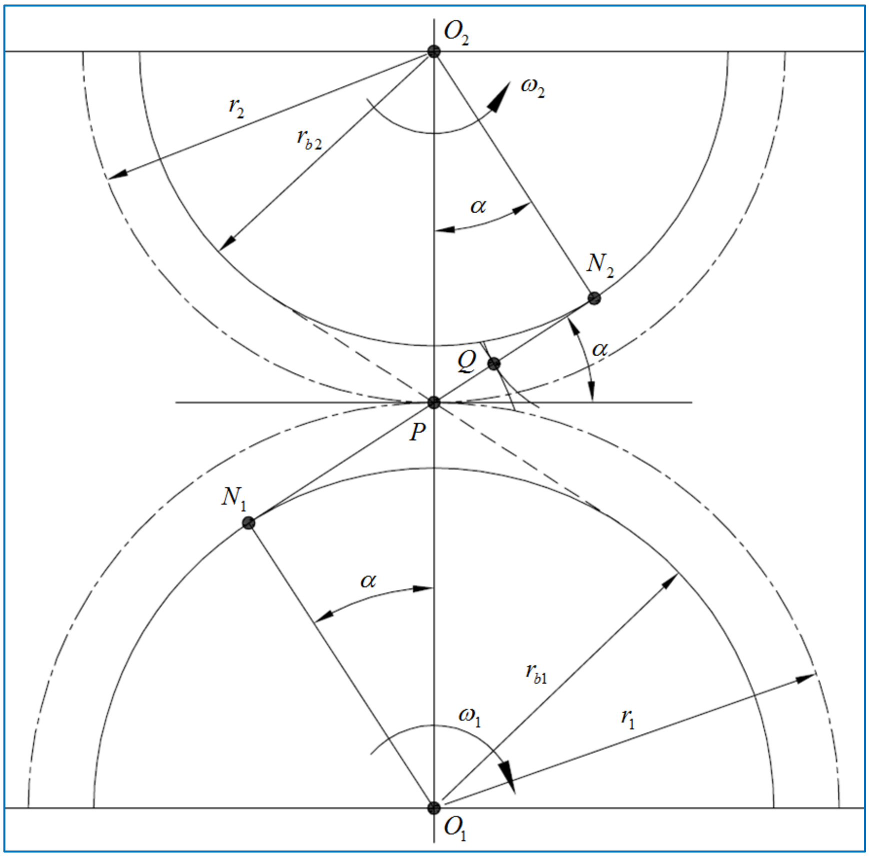
- $r_{b1} d\theta_1 = \overline{C_1C_2} = r_{b2} d\theta_2$
- $r_{b1} \dfrac{d\theta_1}{dt} = r_{b2} \dfrac{d\theta_2}{dt} \Rightarrow r_{b1}\omega_1 = r_{b2}\omega_2$
- $\dfrac{\omega_1}{\omega_2} = \dfrac{r_{b2}}{r_{b1}} = \dfrac{\overline{CO_2}}{\overline{CO_1}}$
- The radius of the base circle of every gear is determined.
- Gears can rotate at a constant speed although the center distance of a pair of Involute gears changes.
- However, the intermeshing pitch circle and pressure angle also change.

3-8. Normal pitch

(1) The distance between an Involute line group on a base circle maintains P_n (Normal pitch)



[Figure 11] Normal pitch



[Figure 12] Pitch and center distance by base circle and pitch circle

- Radius of base circle r_b , diameter d_b

$$p_n = \frac{2\pi r_b}{Z} = \frac{\text{Circumference of base circle}}{\text{Tooth number}} = \frac{\pi d_b}{Z}$$

- Radius of pitch circle r

$$r_b = r\cos\alpha \ , \ d_b = d\cos\alpha$$

- Circular pitch $p = \frac{\pi d}{Z}$

- Normal pitch $p_n = \frac{\pi d_b}{Z} = \frac{\pi d\cos\alpha}{Z} = p\cos\alpha$

- Center distance $A = \frac{d_1 + d_2}{2} = \frac{d_{b1} + d_{b2}}{2\cos\alpha}$

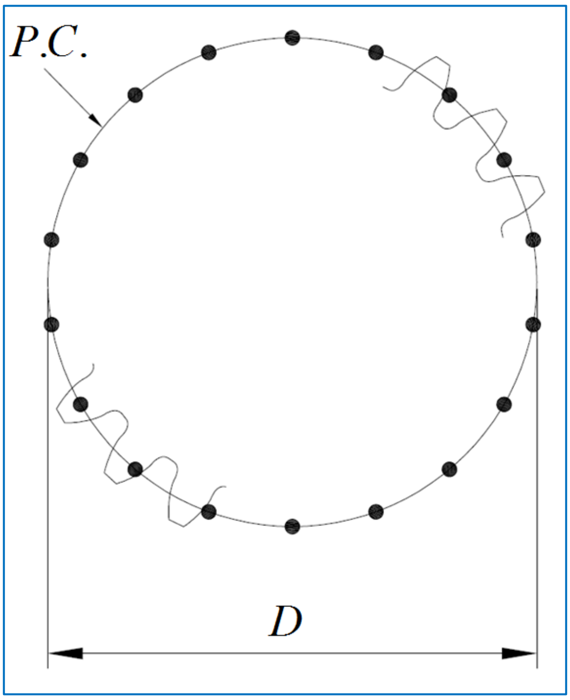
* Standards of gear production : Tooth number → Pressure angle → Module
* Make the same value of module and pressure angle of two engaging gears.

3-9. Size judgement of gear tooth

(1) Although the size of the pitch circle is the same, we can add or remove gear teeth. As such, we need a standard size of gear tooth.

(2) Three standards regarding the size of gear teeth

① Circular pitch : p

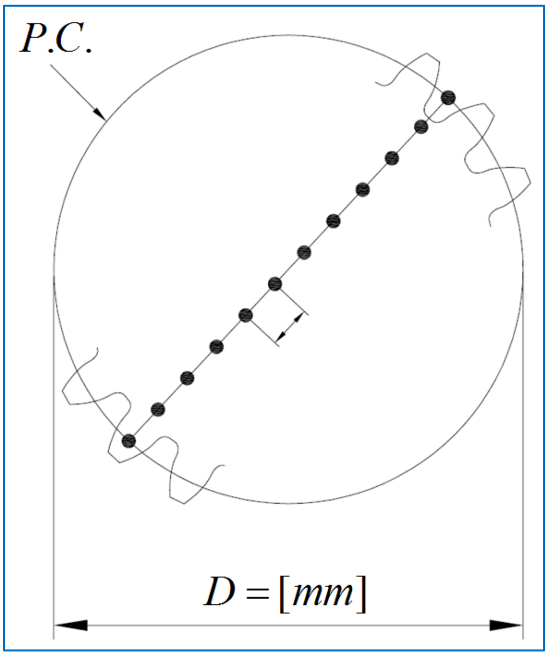


[Figure 13] Size judgement of gear tooth(Judged by circular pitch)

- $P = \frac{\pi D}{Z}$

* The result has a decimal point because of π → As such, it is difficult to calculate.

② Module : m (=addendum)

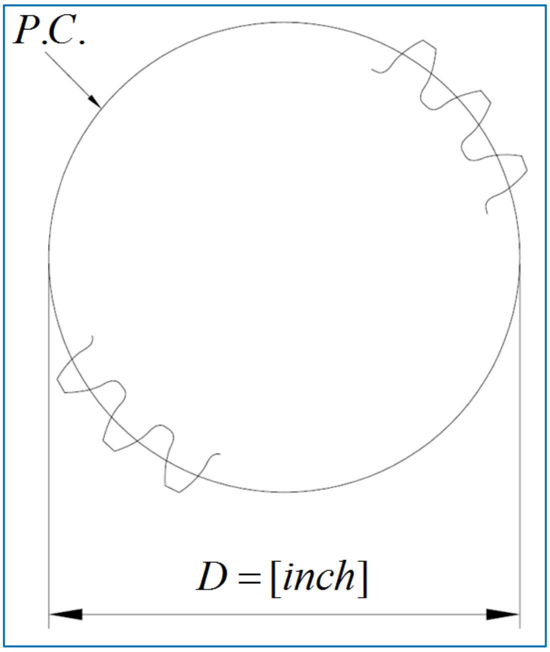


[Figure 14] Size judgement of gear tooth(Judged by module)

- Module $m = \frac{D}{Z} = \frac{\text{Diameter of pitch circle}}{\text{Tooth number}}$

- $\pi m = \frac{\pi D}{Z} = P$

③ Diametral pitch :

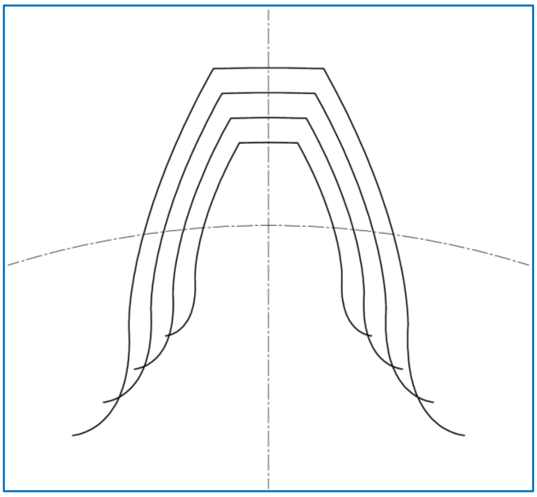


[Figure 15] Side judgement of gear tooth(Judged by diameter pitch)

- $P_d = \frac{Z}{D[\text{inch}]}$

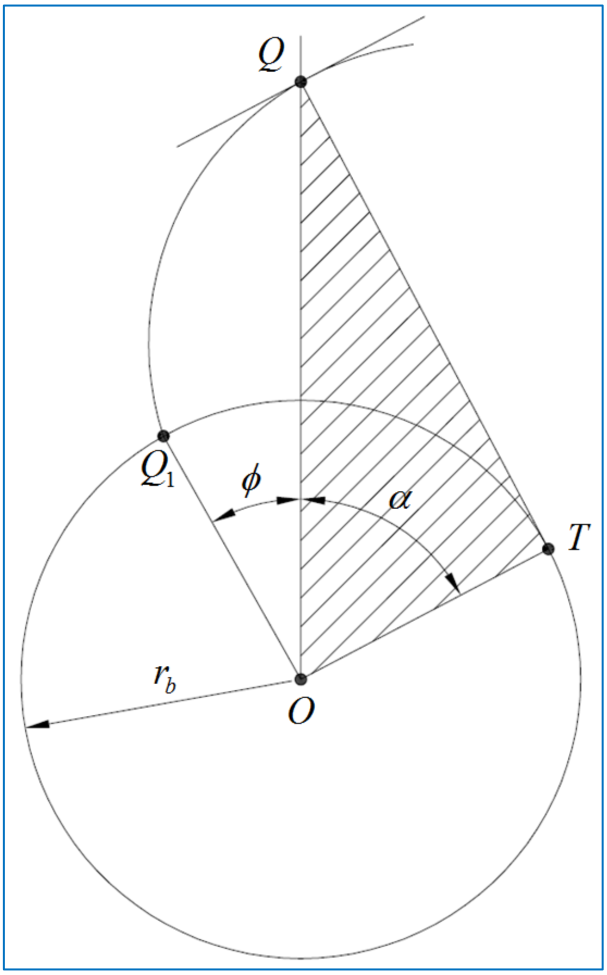
3-10. Normal value of module

(1) If we determine module and diameter pitch as we please, it is inconvenient to design a gear because of a lot of sizes and types of tooth. As such, the normal value of a module and diameter pitch is determined. (ISO, DIN, KS, etc.)



[Figure 16] Various module size

3-11. Involute function

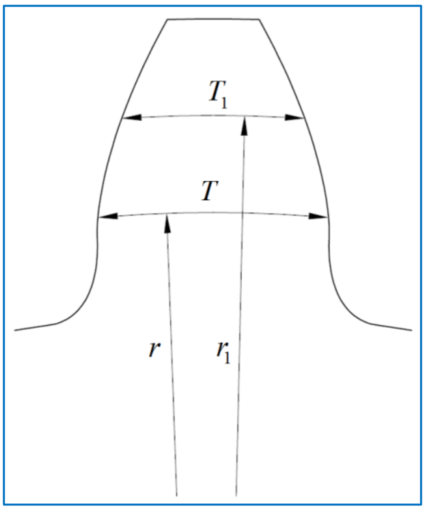


[Figure 17] Involute tooth profile

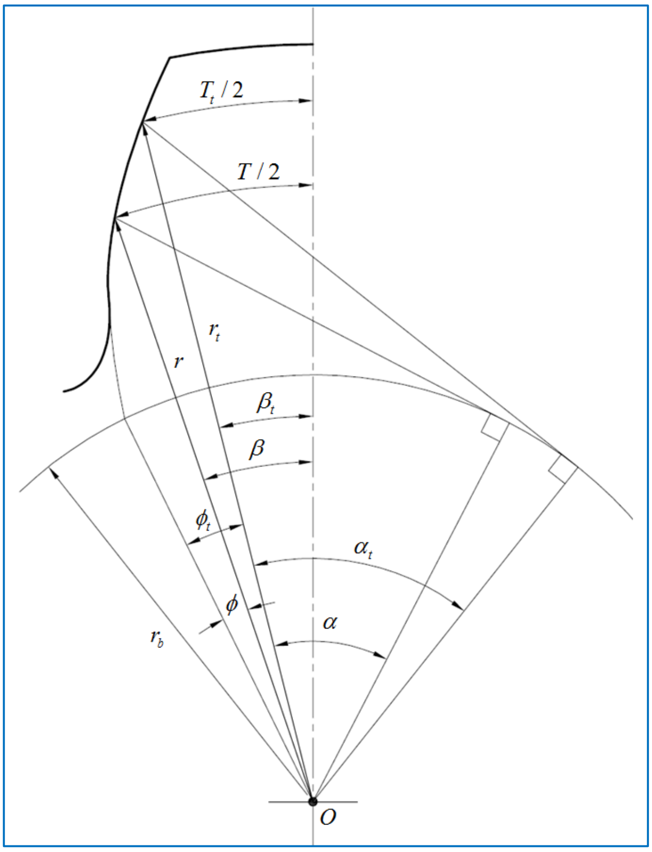
- $\overline{TQ} = \overline{TQ_1}$
- $r_b \tan\alpha = r_b(\phi + \alpha)$
- $\phi = \tan\alpha - \alpha = \text{inv}\alpha\,[\text{rad}]$

* A correlation of a straight line between a point at any involute function and a center of a base circle and of a starting point of an Involute and a contact point as of a straight line.

3-12. Tooth thickness calculation at any radius



[Figure 18] Tooth thickness calculation at any radius



[Figure 19] Tooth thickness

* β , T , r correlations (r_b : radius of base circle , α : base pressure angle)

- $\frac{T}{2} = \beta r$, $\frac{T_t}{2} = \beta_t r_t$
- $\beta = \frac{T}{2r}$, $\beta_t = \frac{T_t}{2r_t}$
- $\beta - \beta_t = \frac{T}{2r} - \frac{T_t}{2r_t}$

In the figure:

$$\beta - \beta_t = \phi_t - \phi$$

$$\beta - \beta_t = \frac{T}{2r} - \frac{T_t}{2r_t} = \phi_t - \phi = \mathrm{inv}\alpha_t - \mathrm{inv}\alpha$$

Arrange the equation by T_t

* $T_t = 2r_t(\frac{T}{2r} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_t) = d_t(\frac{T}{d} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_t)$

We calculate α and α_t using the equation $r_b = r\cos\alpha = r_t\cos\alpha_t$

We can calculate T_t at any radius r_t .

* What is R_t when T_t is 0?

$$\frac{T}{d} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_t = 0$$

$\mathrm{inv}\alpha_t = \mathrm{inv}\alpha + \frac{T}{2r}$ $\rightarrow$ α_t calculation

$r_b = r_t\cos\alpha_t$ $\rightarrow$ r_t calculation

※ It is normally easy to calculate T_t at any radius r_t . But if we want to know r_t when T_t is 0, we need a mathematic technique.

3-13. Standard Spur Gear Correlation

[Table 2] Spur gear correlation

Standard Spur gear correlation		
Tooth number	z	
Module	$m\,(= \frac{d_p}{z} = \frac{p}{\pi})$	
Pressure angle	α	
Clearance	$c \geq 0.25m$	
Addendum	$h_a = m$	
Dedendum	$h_f = m + c \geq 1.25m$	

Whole depth	$h = h_a + h_f \geq 2.25m$	
Working depth	$h_w = h - c = 2m$	
Calculation item		
Diameter of pitch circle	$d_p = mz$	
Diameter of addendum circle	$d_a = d_p + 2h_a = (z+2)m$	
Diameter of dedendum circle	$d_f = d_p - 2h_f = (z-2)m - 2c$	
Diameter of base circle	$d_b = d_p \cos\alpha = mz\cos\alpha$	
Circular pitch	$p = \pi m$	
Normal pitch	$p_n = p\cos\alpha = \pi m\cos\alpha = \frac{\pi d_b}{z}$	
Arc tooth thickness (on pitch circle)	$T_p = \frac{p}{2} = \frac{\pi m}{2}$	
Arc tooth thickness (on base circle)	$T_b = d_b(\frac{T_p}{d_p} + \mathrm{inv}\alpha)$ $= (\frac{\pi}{2} + z\mathrm{inv}\alpha)m\cos\alpha$ ※ $\mathrm{inv}\alpha_b = 0\,(\alpha_b = 0$ at base circle)	
Arc tooth thickness on any cirlcle	$T_t = d_t(\frac{T_p}{d_p} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_t)$	
Limit of sharpness of the edge	$0 = \frac{T_p}{d_p} + \mathrm{inv}\alpha - \mathrm{inv}\alpha_t$	
Displacement over a given number of teeth	$T_m = m\cos\alpha[\pi(z_m - 0.5) + z\mathrm{inv}\alpha]$	
Displacement over a given number of teeth (profile-shifted gear)	$T_m = m\cos\alpha[\pi(z_m - 0.5) + z\mathrm{inv}\alpha] + 2xm\sin\alpha$	
Displacement over a given number of teeth (profile-shifted gear +backlash)	$T_m = m\cos\alpha[\pi(z_m - 0.5) + z\mathrm{inv}\alpha] + 2xm\sin\alpha - \delta\cos\beta\cos\alpha$	
Overpin measurement (even tooth)	$d_m = \frac{mz\cos\alpha}{\cos\phi} + d_p$	
Overpin measurement (odd tooth)	$d_m = \frac{mz\cos\alpha}{\cos\phi} + \cos(\frac{90°}{z}) + d_p$	

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